THE THEORY OF INTERSTELLAR TRADE

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This article extends interplanetary trade theory to an interstellar setting. It is chiefly concerned with the following question: how should interest charges on goods in transit be computed when the goods travel at close to the speed of light? This is a problem because the time taken in transit will appear less to an observer traveling with the goods than to a stationary observer. A solution is derived from economic theory, and two useless but true theorems are proved. (*JEL* F10, F30)

I. INTRODUCTION

Many critics of conventional economics have argued, with considerable justification, that the assumptions underlying neoclassical theory bear little resemblance to the world we know. These critics have, however, been too quick to assert that this shows that mainstream economics can never be of any use. Recent progress in the technology of space travel as well as the prospects of the use of space for energy production and colonization (O'Neill 1976) make this assertion doubtful; for they raise the distinct possibility that we may eventually discover or construct a world to which orthodox economic theory applies. It is obvious, then, that economists have a special interest in understanding and, indeed, in promoting the development of an interstellar economy. One may even hope that formulation of adequate theories of interstellar economic relations will help accelerate the emergence of such relations. Is it too much to suggest that current work might prove as influential in this development as the work of Adam Smith was in the initial settlement of Massachusetts and Virginia?

This article represents one small step for an economist in the direction of a theory of interstellar trade. It goes directly to the problem of trade over stellar distances, leaving aside the analysis of trade within the Solar System. Interplanetary trade, while of considerable empirical interest (Frankel 1975), raises no major theoretical problems since it can be treated in the same framework as interregional and international trade. Among the authors who have not pointed this out are Ohlin (1933) and Samuelson (1947). Interstellar trade, by contrast, involves wholly novel considerations. The most important of these are the problem of evaluating capital costs on goods in transit when the time taken to ship them depends on the observer's reference frame; and the proper modeling of arbitrage in interstellar capital markets where—or when (which comes to the same thing)—simultaneity ceases to have an unambiguous meaning.

These complications make the theory of interstellar trade appear at first quite alien to our usual trade models; presumably, it seems equally human to alien trade theorists. But the basic principles of maximization and opportunity cost will be seen to give clear answers to these questions. I do not pretend to develop here a theory that is *universally* valid, but it may at least have some galactic relevance.

The remainder of this article is, will be, or has been, depending on the reader's inertial frame, divided into three sections. Section II develops the basic Einsteinian framework of the analysis. In Section III, this framework is used to analyze interstellar trade in goods. Section IV then considers the role of interstellar capital movements. It should be noted that, while the subject of this article is silly, the analysis actually does make sense. This article, then, is a serious analysis of a ridiculous subject, which is of course the opposite of what is usual in economics.

II. FUNDAMENTAL CONSIDERATIONS

There are two major features distinguishing interstellar trade from the interplanetary trade we are accustomed to. The first is that the time

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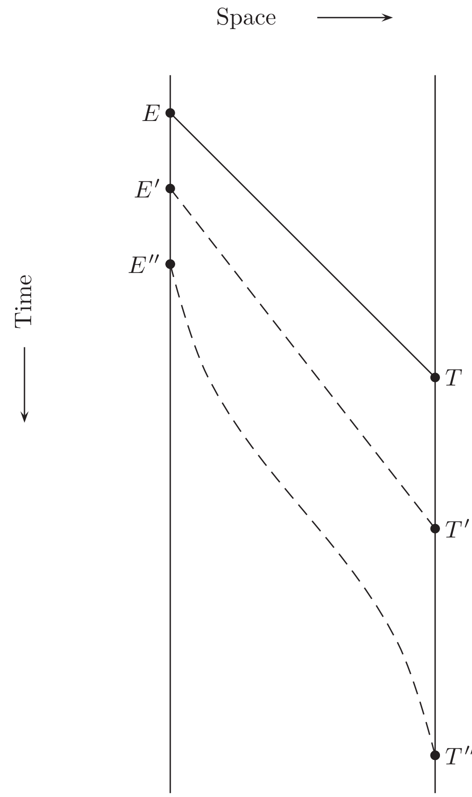
spent in transit will be very great since travel must occur at less than light speed; round trips of several hundred years appear likely. The second is that, if interstellar trade is to be at all practical, the spaceships that conduct it must move at speeds that are reasonable fractions of the speed of light.

Because interstellar trade will take so long, any decision to launch a cargo will necessarily be a very long-term investment project and would hardly be conceivable unless there are very extensive futures markets. I will assume, then, that future futures markets are, well, futuristic in their development. In fact, I will assume that investors, human or otherwise, are able to make perfect forecasts of prices over indefinite periods.

The second feature of interstellar transactions cannot be so easily dealt with (physicists are not as tolerant as economists of the practice of assuming difficulties away). If trading space vessels move at high velocities, we can no longer have an unambiguous measure of the time taken in transit. The time taken by the spacecraft to make a round trip will appear less to an observer on the craft than to one remaining on Earth. Since an interstellar voyage is an investment project that must have a positive present value, there is obviously a problem in deciding which transit time to use in the present value calculation.

This is an inertial problem—which becomes a weighty problem in a gravitational field—requiring an economic analysis, provided in the next section. In this section, I develop the necessary physical concepts, illustrated in Figure 1. Consider trade between two planets, Earth and Trantor. I assume that the two planets may be regarded as being in the *same inertial frame*. Then their world lines in space-time can be represented by two parallel lines, shown as $EE'E''$ and $TT'T''$ in the figure. Several types of contact between the two planets are also shown. The line ET is the world line of an electromagnetic signal—say, a rerun of *Star Trek*—sent from Earth to Trantor. If time is measured in years and space in light years, ET will have a 45° slope. The line $E'\,T'$ is the world line of a spaceship moving with uniform velocity from Earth to Trantor. It must be steeper than ET because the spaceship's speed must be less than that of light. Finally, $E''T''$ shows a spaceship path, which is more likely in practice: it involves initial acceleration, followed by deceleration.

FIGURE 1
World Lines in Space-Time between Earth and Trantor



The problem of time dilation must now be considered. It will suffice here to consider the case of a spaceship with uniform velocity. It is then well known—see, for example, Lawden (1962)—that if the voyage from Earth to Trantor appears to take n years to observers in the Earth-Trantor inertial reference frame, it will appear to take $\overline{n}$ years aboard the spaceship, where:

$$\overline{n} = n\sqrt{1 - \frac{v^2}{c^2}}, \tag{1}$$

where v is the spacecraft's velocity and c the speed of light. This can easily be demonstrated by representing the voyage in Minkowski space-time, that is, with a real space axis and an imaginary time axis. The ship's velocity can then be represented by a rotation of the axes; the rotation of the time axis is shown in Figure 2. (Readers who find Figure 2 puzzling should recall that a diagram of an imaginary axis must, of course, itself be imaginary.)

FIGURE 2
Rotation of the Imaginary Time Axis in Minkowski Space-Time

To conclude this section, we should say something about the assumption that the trading planets lie in the same inertial frame. This will turn out to be a useful simplification, permitting us to limit ourselves to consideration of special relativity. It is also a reasonable approximation for those planets with which we are likely to trade. Readers may, however, wish to use general relativity to extend the analysis to trade between planets with large relative motion. This extension is left as an exercise for interested readers because the author does not understand the theory of general relativity, and therefore cannot do it himself.

III. INTERSTELLAR TRADE IN GOODS

We are now prepared to begin the economic analysis. Let us start with some notation. Let

p_E, p_T = price of Terran, Trantorian goods on Earth

p_E^*, p_T^* = price of Terran, Trantorian goods on Trantor

r, r^* = interest rates on Earth, Trantor

N = number of years taken to travel from Earth to Trantor (or vice versa), as measured by an observer in the Earth-Trantor inertial frame.

All these quantities except N should, of course, be defined at a point in time; except where specified, however, I will make the simplifying assumption that these quantities are in fact constant over time.

Now let us begin by considering the simplest kind of interstellar transaction, one which will reveal the problems of analysis and also give us the key to their solution. Suppose a Trantorian merchant decides to consider trading with Earth. Assume, provisionally, that interest rates are the same on both planets. (This assumption will be justified in the next section.) Then, it (the merchant) may have in its mind (or equivalent organ) a series of transactions of the following kind. It will make an initial expenditure of $c + q_T^* p_T^*$, where c is the cost of outfitting a ship and q_T^* is the quantity of Trantorian goods shipped. When the ship reaches Earth, the goods will be exchanged for a quantity of Earth goods; given the notation already developed, this quantity will be $q_E^* = \frac{q_T^* p_T}{p_E}$. Finally, on return, the goods will be sold at the price p_E^*, yielding revenue $\frac{q_T^* p_T p_E^*}{p_E}$.

Is this transaction profitable? A merchant staying home on Trantor will ask whether the present value of the revenue exceeds the initial cost; since the trip takes $2N$ years from the point of view of a stationary observer, the test criterion is:

$$(2) \qquad \frac{q_T^* p_T p_E^*}{p_E} \geq (c + q_T^* p_T^*)(1 + r^*)^{2N}.$$

But suppose the merchant had traveled with its cargo? The trip would then, from its point of view, have taken only $2N\sqrt{1 - \frac{v^2}{c^2}}$ years, suggesting an alternative criterion of acceptance,

$$(2') \quad \frac{q_T^* p_T p_E^*}{p_E} \geq (c + q_T^* p_T^*)(1 + r^*)^{2N\sqrt{1 - \frac{v^2}{c^2}}}$$

These criteria cannot both be right. Which, then, is correct?

The answer may be obtained by considering the justification for present value calculations. A present value calculation makes sense because it takes account of opportunity cost: an investor might, instead of undertaking a project, have bought a bond. In this case, the merchant might have bought a bond *on Trantor* and let it mature instead of sending a cargo to Earth. The value of the bond on the ship's return does not depend on the time elapsed on board the ship itself. So Equation (2), not Equation (2′), is the proper criterion. We have thus demonstrated the following.

> **First Fundamental Theorem of Interstellar Trade:** *When trade takes place between two planets in a common inertial frame, the interest costs on goods in transit should be calculated using time measured by clocks in the common frame and not by clocks in the frames of trading spacecraft.*

At this point, it is unlikely that the reader will raise the following objection. Suppose that the merchant, instead of making a round trip, were to travel with its cargo and settle down on Earth as a rich . . . well, not man, but say a rich being. Would the argument still be valid?

We can most easily see that the argument is still valid if we consider a special case. Suppose that the transportation costs other than interest on goods in transit are negligible; and suppose further that the interstellar shipping industry is competitive, so that profits are driven to zero. Then, if Equation (2) is a correct criterion we have the relationship:

$$\frac{p_E^*}{p_T^*} = \left(\frac{p_E}{p_T}\right)(1 + r^*)^{2N}. \tag{3}$$

Thus, relative goods prices will not be equalized; rather, there will be a wedge driven between relative prices on Earth and on Trantor.

Now, within this special case, consider the position of a Trantorian planning to migrate to Earth. It could purchase a cargo on Trantor and sell it on Earth. Alternatively, though, it could buy a bond on Trantor and, on reaching Earth, sell its claim to a Terran planning to travel in the opposite direction. Because of this alternative possibility, the fact that the merchant itself does not plan to make a round trip is inessential since what the Terran will be willing to pay for the claim will reflect the extent to which its value will have grown on Trantor when the Terran arrives. A one credit (Trantorian) bond, bought by a merchant just about to migrate, will have grown in value to CrT.$(1 + r^*)^{2N}$ by the time a migrant in the other direction can arrive to claim it. Such a migrant would have the choice of buying the bond or carrying Earth goods with him, so arbitrage will mean that the price of the claim on Earth will be CrE.$(1 + r^*)^{2N}\left(\frac{p_E}{p_E^*}\right)$.

But one credit (Trantorian) worth of cargo shipped from Trantor to Earth will sell for CrE.$\left(\frac{p_T}{p_T^*}\right)$, which by Equation (3) is equal to CrE.$(1 + r^*)^{2N}\left(\frac{p_E}{p_E^*}\right)$. So the Trantorian merchant will be indifferent between shipping goods and buying a bond. This shows that the First Fundamental Theorem of Interstellar Trade remains valid, even if no spacecraft or individuals make round trips. All that is necessary is that there be two-way trade, with somebody or something going in each direction.

This proof has been for a special case; but the proposition is in fact relatively general. (The reader must, of course, be careful not to confuse relative generality with general relativity.) A proof of the First Fundamental Theorem in the presence of transportation costs may be found in an unwritten working paper by the author (Krugman 1987).

IV. INTERSTELLAR CAPITAL MOVEMENTS

Alert readers will have noticed that the analysis of interstellar trade in goods already involves some discussion of asset markets, both because interstellar transportation costs depend on interest rates and because the validity of the First Fundamental Theorem depends on arbitrage through interspecies transactions in securities. Further, the results of the last section depended on the assumption of equal interest rates on the two planets. In this section, we will examine the effects of interstellar capital movements. In particular, we want to know whether interstellar arbitrage will in fact equalize interest rates.

One might at first doubt this. Arbitrage is possible internationally because an investor can choose between holding his wealth in different countries for the next, say, 30 days simply by calling up his broker and instructing him. In interstellar trade, things are not so simple. Even if we leave on one side the problem that nonhuman brokers may not have ears, let alone telephones, there is the problem that simultaneous arbitrage is not possible. Messages must travel at light speed; goods more slowly still. We have already seen that this means that relative goods prices will vary from planet to planet, even if there are no transportation costs in the usual sense. Will not interest rates differ as well?

Perhaps surprisingly, the answer is no. It will suffice to consider a particular example of an interstellar capital transaction. Suppose that, as in the last section, interest costs on goods in transit are the only transportation costs. Then, Equation (3) will hold for relative prices. Now let a Trantorian resident carry out the following set of transactions: (1) it ships goods to Earth; (2) it then invests the proceeds from selling these goods in Terran bonds for K years; (3) it then buys Terran goods and ships them to Trantor. The return on this set of transactions, viewed as an investment, must be the same as the return on holding bonds for the same period, that is, $2N + K$ years. This gives us the condition:

$$(4) \quad (1 + r^*)^{2N+K} = \left(\frac{p_E^*}{p_T^*}\right)\left(\frac{p_T}{p_E}\right)(1 + r)^K.$$

But if we use relationship (3), this reduces to $r = r^*$. We have thus arrived at the result that interest rates will be equalized.

> **Second Fundamental Theorem of Interstellar Trade:** *If sentient beings may hold assets on two planets in the same inertial frame, competition will equalize the interest rates on the two planets.*

Combining the two theorems developed in this article, it will be seen that we have the foundation for a coherent theory of interstellar trade between planets in the same inertial frame. Interstellar trading voyages can be regarded as investment projects, to be evaluated at an interest rate that will be common to the planets. From this point, the effects of trade on factor prices, income distribution, and welfare can be traced out using the conventional tools of general equilibrium analysis. The picture of the world—or, rather, of the universe—which emerges is not a lunatic vision; stellar, maybe, but not lunatic.

Is space the Final Frontier of economics? Certainly this is only a first probe of the subject, but the possibilities are surely limitless. (In curved space-time, of course, this does not prevent the possibilities from being finite as well!) I have not even touched on the fascinating possibilities of interstellar finance, where spot and forward exchange markets will have to be supplemented by conditional present markets. Those of us working in this field are still a small band, but we know that the Force is with us.

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